Experiments in the wind turbine far wake for the evaluation of analytical wake models

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Introduction

• Wakes in wind farms:
  – Lower efficiency
  – Fluctuating loads

Horns Rev offshore wind farm west of Denmark. Credit: Vattenfall
Introduction

• Important to know the evolution of the parameters
Velocity decay & Width development
Analytical model

• Schlichting model
  – Blunt bodies

• Objective
  – Wind turbines
Parameters

- **Velocity Decay**
  \[ U_1 = U_\infty - U \]
- **Width development**
  - Half depth width
  \[ b_{1/2} = f(x, U_1) \]
Analytical description of wakes

- Analytical expressions in terms of the downstream distance.

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<th>Velocity decay</th>
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<td>Circular wake</td>
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$$\frac{b_{1/2}}{D} = C_b \cdot \left(\frac{x}{D}\right)^{1/3} \quad \quad \frac{U_1}{U_\infty} = C_u \cdot \left(\frac{x}{D}\right)^{-2/3}$$

- Velocity distribution

$$\frac{u_1}{U_\infty} = \frac{\sqrt{10}}{18\beta} \cdot \left(\frac{x}{C_T A}\right)^{-2/3} \cdot \left\{1 - \left(\frac{z}{b}\right)^{3/2}\right\}^2$$
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Experimental setup

• Instrumentation
  – Wind tunnel
  – Laser Doppler Velocimeter
  – Turbine

• Full 2-D wakes
• Lines wakes
• 2 turbulence ambient conditions
Results of mean velocity 2 D full wakes

- Downshift of the wake due to tower effect [1]

Results of mean velocity in the $xz$ plane

- Expansion of the wake
- Decrease of the velocity decay
Discussion of the velocity decay

Velocity decay Low TI

- Deviation of 2.0%

\[
\frac{U_1}{U_\infty} = C_u \cdot \left( \frac{x}{D} \right)^{-2/3}
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Velocity decay High TI

- Deviation of 2.5%
Discussion of the width development

Width development Low TI

- Deviation of 2.5%

\[
\frac{b_{1/2}}{D} = C_b \cdot \left(\frac{x}{D}\right)^{1/3}
\]

Width development High TI

- Deviation of 3.0%
Discussion of the velocity distribution

- High degree of agreement
- Some deviations in the edges

\[
\frac{u_1}{U_\infty} = \frac{\sqrt{10}}{18\beta} \cdot \left( \frac{x}{C_T A} \right)^{-2/3} \cdot \left\{ 1 - \left( \frac{z}{b} \right)^{3/2} \right\}^2
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Turbulence intensity
Turbulence intensity models

• Three different models
  – Crespo and Hernandez
  – Quarton
  – Frandsen and Thogersen

• Input parameters

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</tr>
<tr>
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<td>D, $V_{hub}$</td>
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Results of turbulence intensity in the xz plane

- Decrease of the added turbulence intensity
Discussion of turbulence intensity

• Good agreement of the measurements with the models
Conclusion

• Investigation of the Schlichting model for wind turbines
• Good agreement between experiments and theory
• Gaussian shape prediction
• Fairly well prediction of the turbulence intensity of the wake

• Analytical models can describe the wake characteristics
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Thank you for your attention, Questions, comments?